What Is Math Rigor?

It May Not Mean What You Think It Means

An analysis of the meaning of rigor as framed by the learning expectations presented in new math standards and assessments.

A Publication by

MIND Research Institute
A neuroscience and education social benefit organization

mindresearch.org

TABLE OF CONTENTS

INTRODUCTION

The Problem With Rigor

Consider This Scenario:

Maria's favorite subject in school has always been math. Any problem her teacher gives her, she can solve it. She once solved 50 problems in one sitting! It feels good to know the answers without having to think too hard. On the day Maria takes the new standardized test for math, she's confident and excited to show off her knowledge.

Unfortunately, Maria's excitement doesn't last long. She comes to a problem that confuses her. It seems to be asking about something similar to what she's done in class, but there are no numbers to plug into the formulas she knows. She doesn't know how to approach this problem and many of the ones that come after. She leaves the test feeling discouraged and downhearted. Her confidence is diminished and she stews for the rest of the week about how badly she did.

What happened? Maria's testing experience reveals an all too familiar **disconnect between classroom math education and the expectations of state assessments**. If we look at a common dictionary definition of the word *rigor*, we can gain some insight into how this disconnect came about:

Rigor, (n.) \ri-ger\:
a condition that makes life difficult, challenging, or uncomfortable. The quality of being unyielding or inflexible. ***Synonyms****: hardship, adversity, hardness, difficulty.*

(Source: Merriam-Webster)

> *Only about 44 percent of U.S. high school graduates in 2013 were considered ready for college work in mathematics, as measured by ACT and SAT scores. … The need for coherent standards that promote college and career readiness has been endorsed across all states and provinces …*
>
> **NCTM**
> *Principles to Action: Ensuring Mathematical Success for All*

A Call for Change

For years, reformers have called for a shift toward more rigorous educational standards and assessments. The writers of the new standards intend for rigor to yield deeper understanding, and develop students' perseverance and creative problem-solving abilities, which will ultimately prepare them for future college and career demands.

But in practical application, schools often interpret rigor in a way that uses procedural memorization and repetition as the primary learning means. And in many cases, the quest for rigorous education has translated to arduous, inflexible, and harsh learning experiences.

Likely, Maria's teacher would agree that this kind of environment is inconsistent with the overarching goal of creating a motivational and academically inquisitive atmosphere. Like many educators, she is searching for a way to implement rigor in the classroom without making students feel *rigor mortis* (medical definition: stiff and dead)!

And so the question remains, how do we equip our students to be independent, creative problem solvers when facing rigorous material, without relying on rigid pathways or stock formulas as the only means to solving challenging problems?

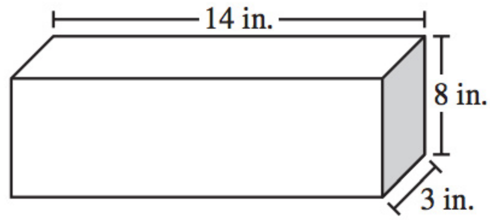
The answer to this impasse is clear: **we need a new shared understanding of rigor**. We need to bridge the disconnect between classroom learning and standardized tests. To do this, gaining a better understanding of the kind of thinking and problem solving (i.e., rigor) called for in the new assessments is a good place to start.

CHAPTER 1

The Way It Used to Be

A Look at the Past

To begin reframing our understanding of mathematical rigor, let's take a look at why even bright, hard working students like Maria are so confused by the new assessments. Here's a sampling of the kind of volume problems students were used to seeing on math assessments.



This rectangular prism has a length of 14 inches, a height of 8 inches, and a width of 3 inches. What is the volume?

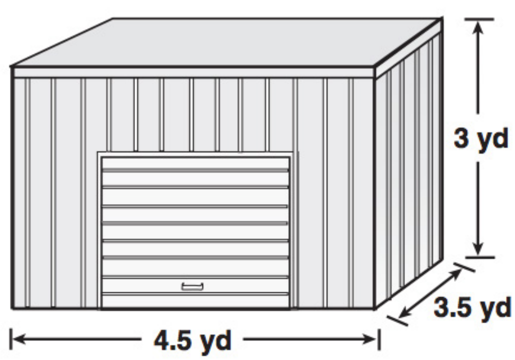
A 25 cu in.

B 42 cu in.

C 112 cu in.

D 336 cu in.

What is the volume, in cubic yards, of the storage unit below?



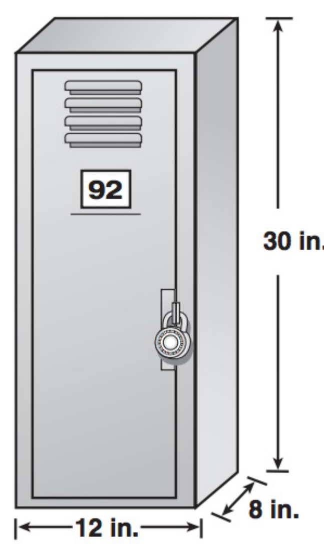
A 11

B 24

C 40.5

D 47.25

What is the volume, in cubic inches, of the school locker below?



A 2880

B 2580

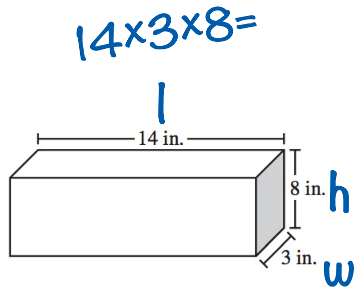
C 390

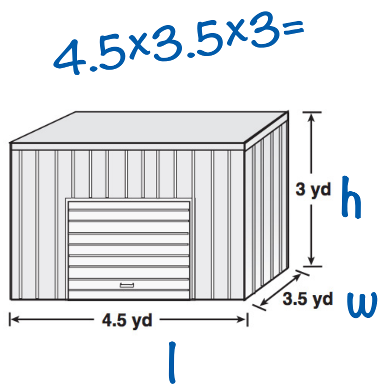
D 360

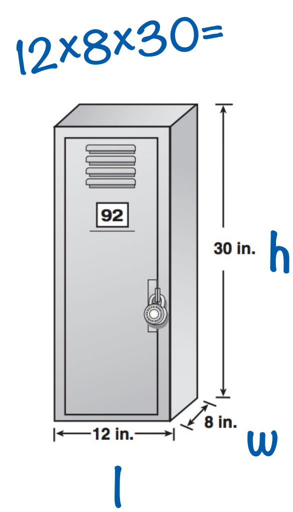
Source: Grade 5 from the California Standards Test (California Dept. of Ed)

Notice that these problems can all be solved by plugging in the three given numbers into the volume formula, volume = length × width × height (v=lwh).

Apart from minor differences, each task is largely the same. There is one clear pathway to each solution and all students need to do is **memorize the steps and repeat them** over and over.







But, what happens when you present the same problem to students in a different way?

As with Maria, many students have not gained a conceptual understanding of the math behind problems like these. They may have the test item structure and formula firmly memorized, but the writers of the new assessments stress that this is not the road to equipping mathematical problem solvers.

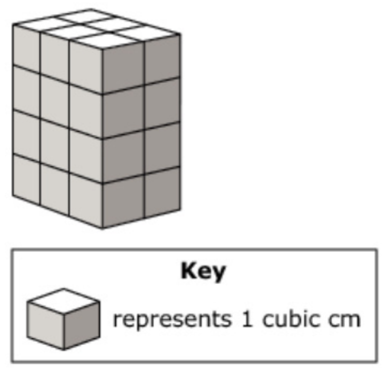
> *Memorization as a learning strategy may work with easy problems, but it is unlikely to be effective if it is the only strategy used when confronted with complex mathematics problems.*
>
> *The Organization for Economic Cooperation and Development (OECD)*

CHAPTER 2

The Way It Is Now

Now let's take a look at the kind of questions Maria saw on the new assessment.

The rectangular prism shown has 4 layers with 6 cubes in each layer.



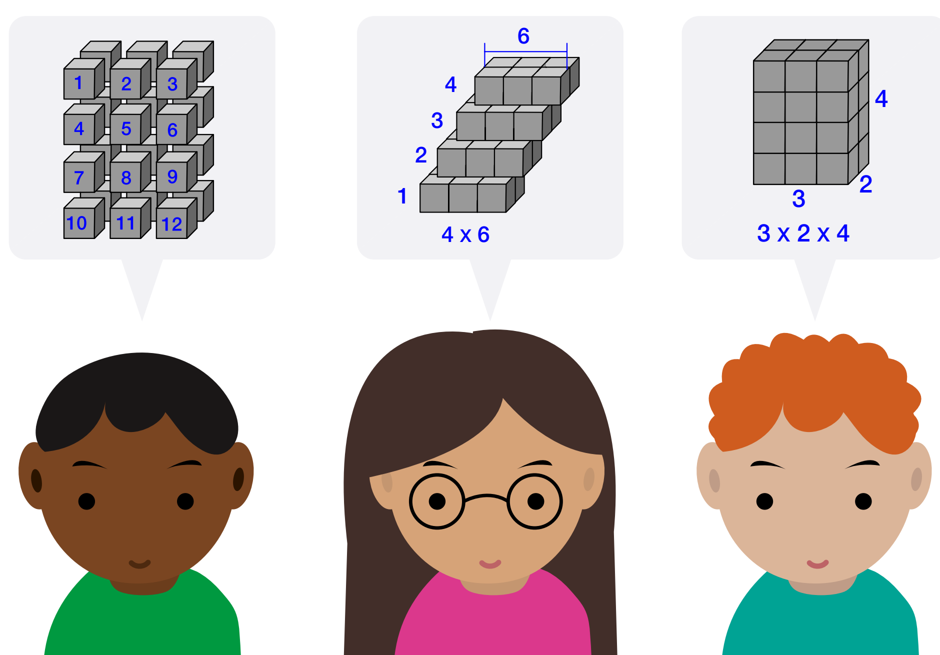
Enter the volume, in cubic centimeters, of the rectangular prism.

Source: Sample grade 5 math problem from Smarter Balanced Assessment Consortium.

Think, for a second, about why a question like this might challenge some students' understanding of volume. First, the problem requires **conceptual understanding** of volume as a quantity of space measured in three-dimensional units. It purposefully steers students away from thinking about volume as just a formula (v=lwh).

Second, this question allows for **creative problem solving**. There isn't one clear pathway to the solution, which may leave some students feeling lost. The problem encourages students to take their own unique pathway to finding the answer.

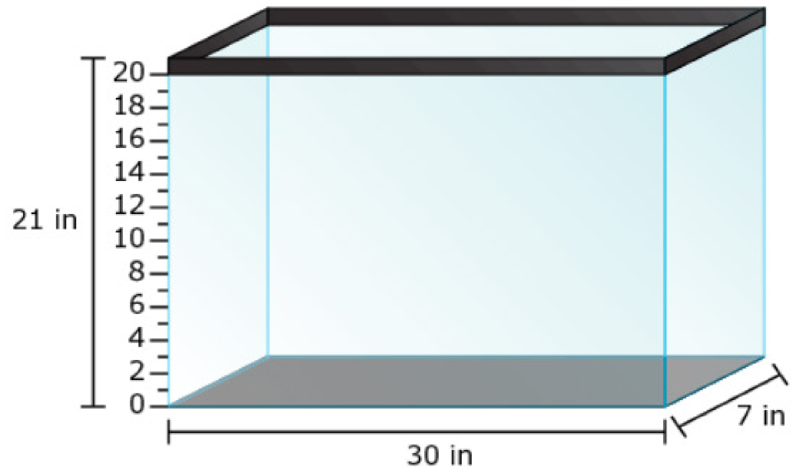
To solve the problem, students must have a conceptual understanding of the question and then employ their own creative solution. Here are three ways to visualize the solution to the problem.



Another problem on the new state test looks like this:

Walter puts 1050 cubic inches of dirt into the tank shown.

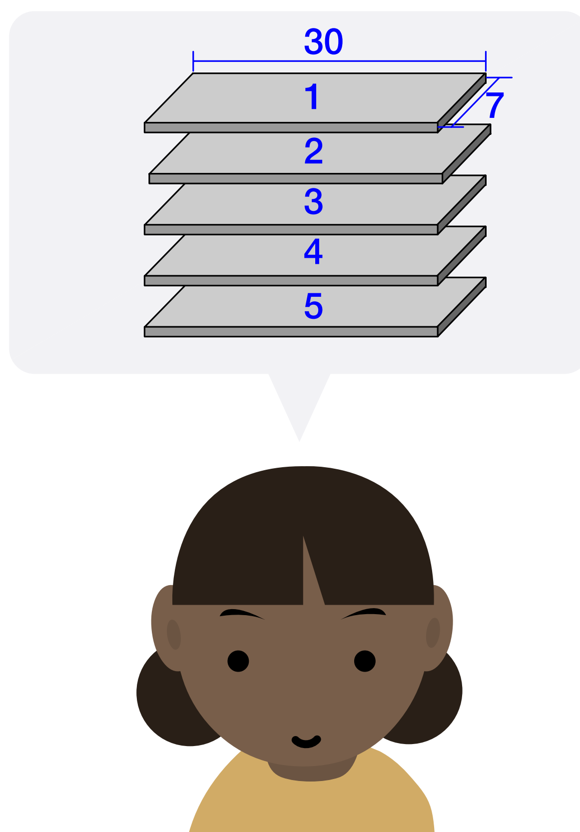
Click the number line to show the height of the dirt in the tank.



Source: Sample grade 5 math problem from Smarter Balanced Assessment Consortium.

Notice this question is assessing the same concept as before, but the given information is completely different. Students must figure out what information is important in order to find the height. This problem asks for conceptual understanding to be applied in a very different (or novel) context.

When students have internalized how the length, width and height of a 3-dimensional shape affect its volume, they are then equipped to approach volume problems from different perspectives. One way of thinking about this problem conceptually is to visualize 30 x 7 rectangular slats of dirt and ask yourself how many of these are needed to fill 1050 cubic units of the tank. The practice of visualizing the steps to the solution is necessary as problems become more and more complex.



CHAPTER 3

A New Understanding of Rigor

> *Students have the competence to think in complex ways and apply knowledge and skills they have acquired.*
>
> **Willard R. Daggett,** *Rigor/Relevance Framework*

Careful analysis of new assessments and standards lead to some important observations about the kind of educational rigor that develops creative and persistent problem solving.

A New Understanding of Rigor:

1. **Rigor guides students toward applying the highest levels of thinking.** It moves them past simple recall of facts or procedures and prompts them to investigate, pull from existing conceptual and procedural knowledge, and create their own solution.

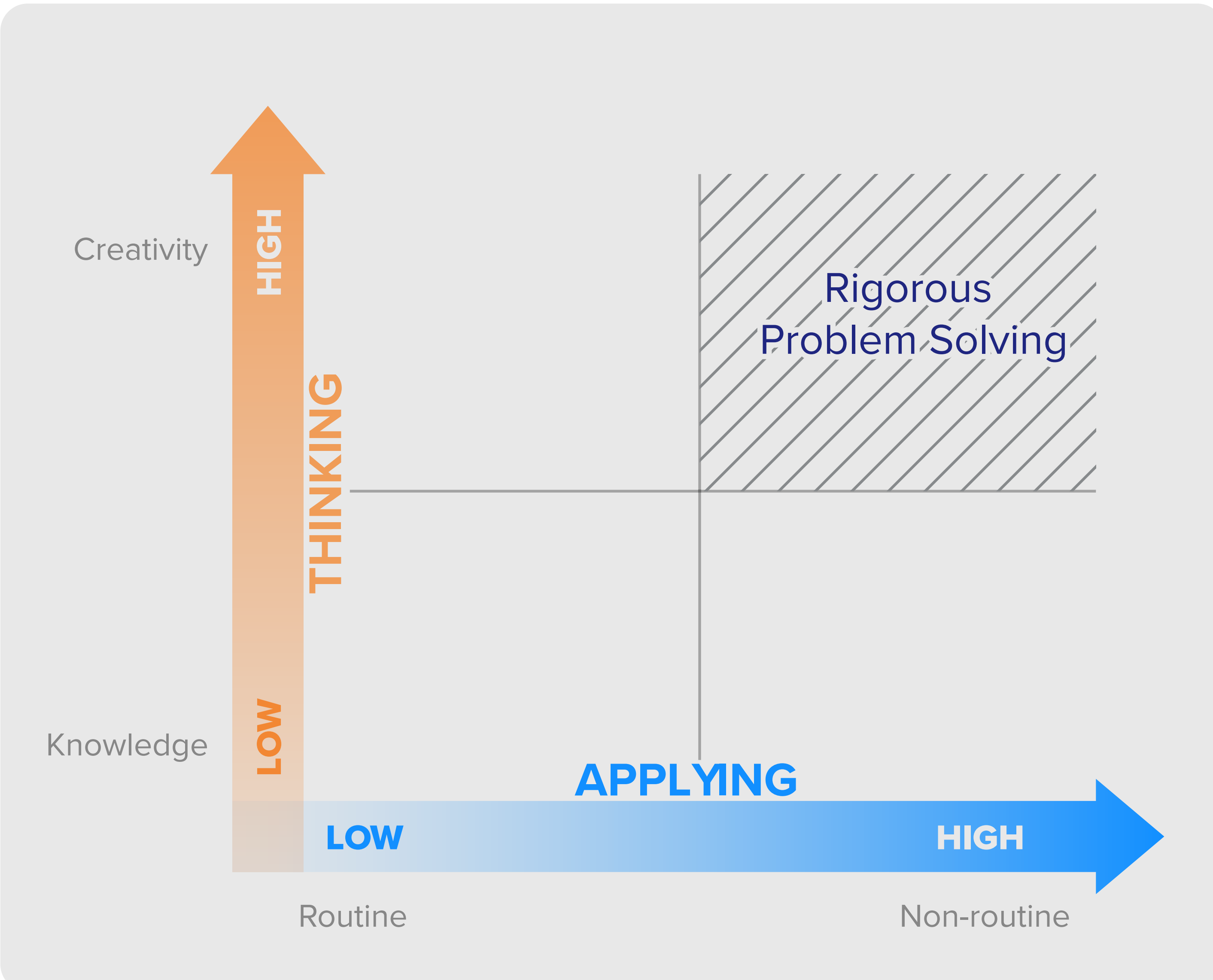
2. **Rigor asks students to apply what they have learned in non-routine situations.** Not only will students regularly engage in higher-order thinking, but they also must be able to do this in any context.

3. **Rigor encourages students to be bold and fearless when tackling difficult problems.** Perhaps one pathway may not lead to the answer, but it may open doors to ultimately finding a solution.

4. **Rigor should produce intrinsic motivation for students.** The positive experience of persevering through difficult problems develops an internally conditioned response to tackle the next problem with energy and enthusiasm.

Rigorous Problem Solving

The goal of rigorous math instruction is to guide students toward becoming highly capable creative problem solvers in non-routine situations.



CHAPTER 4

Guiding Principles

Rigorous instruction must move students to a place where they can apply creative problem solving in complex situations.

Guiding Principles for Increasing Rigor in Classrooms:

1. **Make Learning Accessible** - Learning activities should be accessible regardless of prior knowledge, language proficiency, learning differences, backgrounds, etc. By removing these barriers, we provide all students an equal opportunity to build their aptitudes for rigorous learning.

2. **Encourage Learning from Mistakes** - Students must feel confident to try an approach even though it may fail to lead to the answer. This process opens new insights that will ultimately lead the student to finding the solution to the problem in their own way.

3. **Inspire Creativity** - As much as possible, give students problems for which there are many ways to find the answer. Open-ended problems encourage students to forge their own paths and find their own solutions.

4. **Build Conceptual Understanding** - Understanding symbols, algorithms and procedures is important, but building meaning behind them is crucial for true mastery. Conceptual understanding is the fuel students will need for success with rigorous instruction.

5. **Facilitate Transference** - Give students opportunities to take what they have mastered and apply it in multiple different situations. Non-routine problems encourage students to expand their thinking, make connections and ask new questions.

> *The important thing is not to stop questioning. Curiosity has its own reason for existing.*
>
> **Albert Einstein**

CHAPTER 5

The Pathway to Rigor

At its best, rigorous instruction is a well-oiled machine, simultaneously prompting higher-order thinking and building new knowledge to be applied in the next task. This does not happen in a rigid environment that leaves students feeling anxious and uncomfortable. On the contrary, students need learning experiences that promote mathematical curiosity and the freedom to learn from mistakes.

Here's Our Redefined Understanding of Math Rigor:

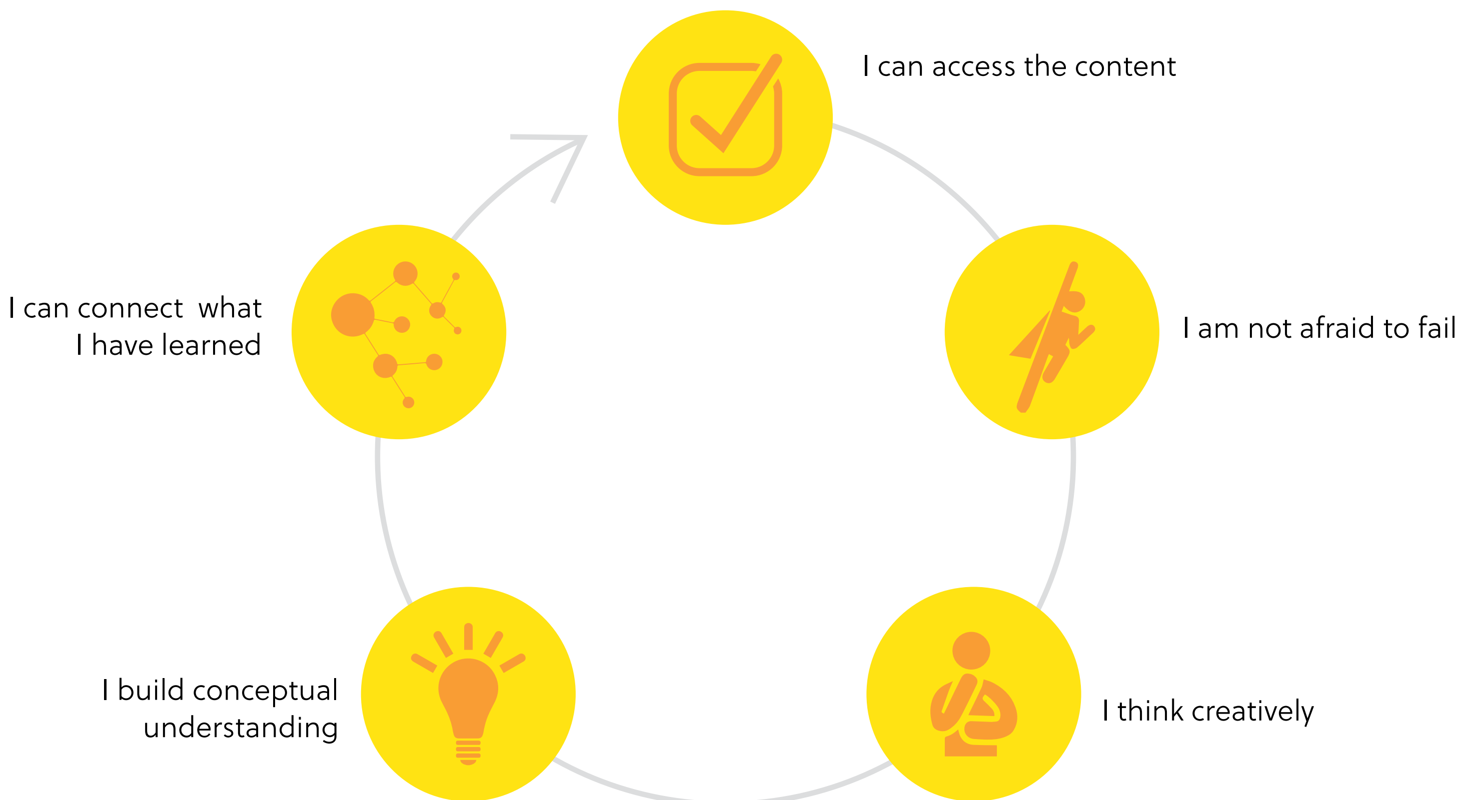
Rigor, (noun), \ri-ger\:

Educational Definition - A condition that encourages creative problem solving in new, exciting contexts, yielding perseverance and satisfying growth in knowledge and skills.

Synonyms: flexibility, progress, creativity, challenge, growth, inspiration.

The Cycle of Rigorous Learning

When students embody the characteristics of rigorous learning, it becomes a cycle of growth and continues to develop their capacity for deeper thinking and increasingly complex problem solving.



Rigorous problem solving should always be challenging. But given the right opportunities, students may just come to enjoy it! Rigor takes students out of their automatic framework of thinking, pushing them beyond the threshold of what they already know. It moves them into creative and effortful problem solving. It is there that they expand their capacity for deeper understanding. As students find success with rigorous instruction, they develop a thirst for challenge and any limitations they face in problem solving begin to fade away.

RIGOR moves us into **creative & effortful problem solving**

CHAPTER 6

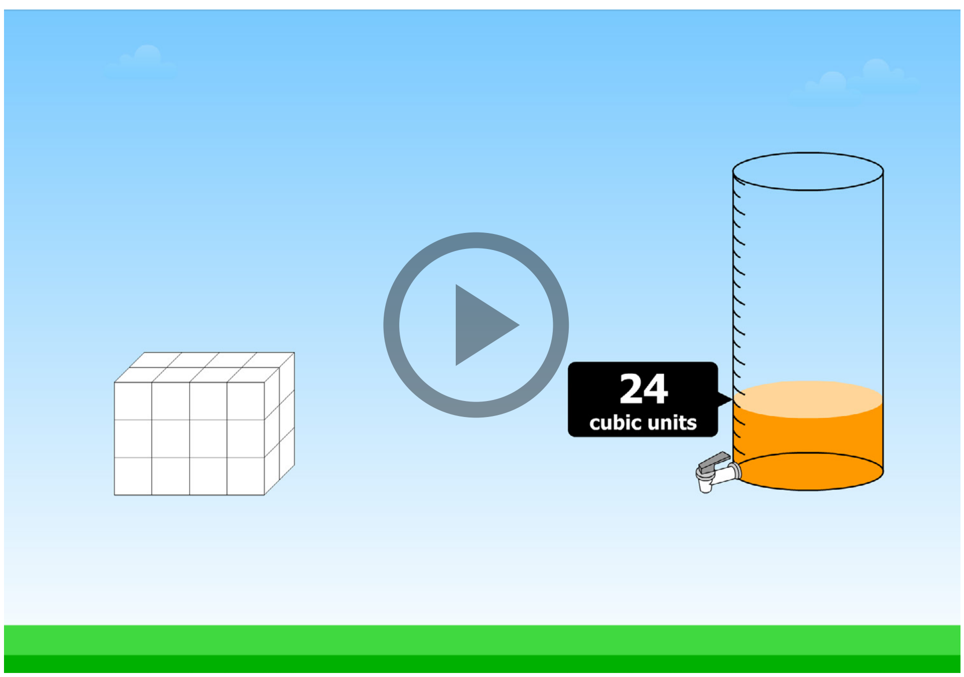
The Case of ST Math

ST Math Volume Games: Rigorous Learning Through Play

Here are some examples of how the ST Math K-12 instructional program moves students to rigorous problem solving. ST Math leverages visual, game-based learning to build conceptual understanding of volume while mastering content knowledge. Let's explore how these interactive puzzles employ the guiding principles we've covered.

Making Learning Accessible

These puzzle sequences allow students with limited English proficiency or prior content knowledge to engage in the problem solving. The first puzzle sequence below visually asks the question: **How many cubic units of liquid will fill the box?**

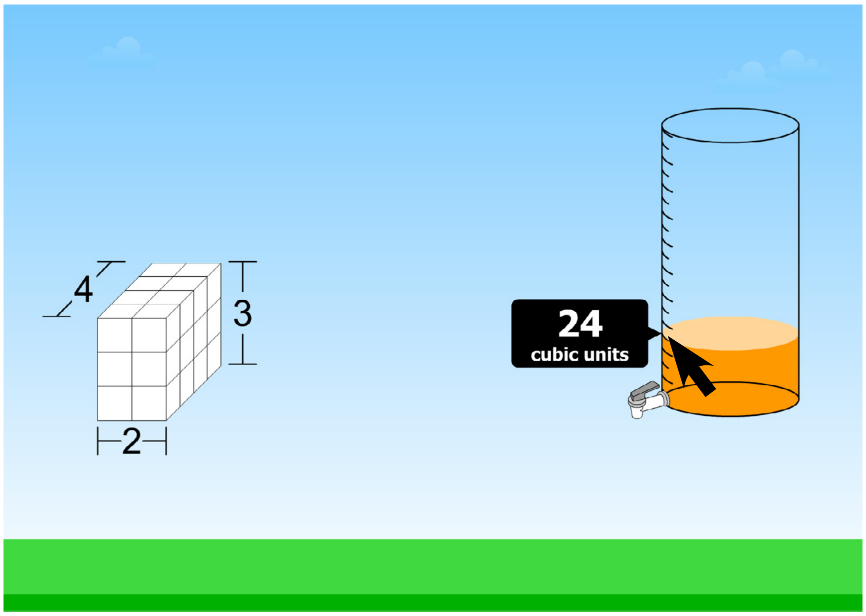


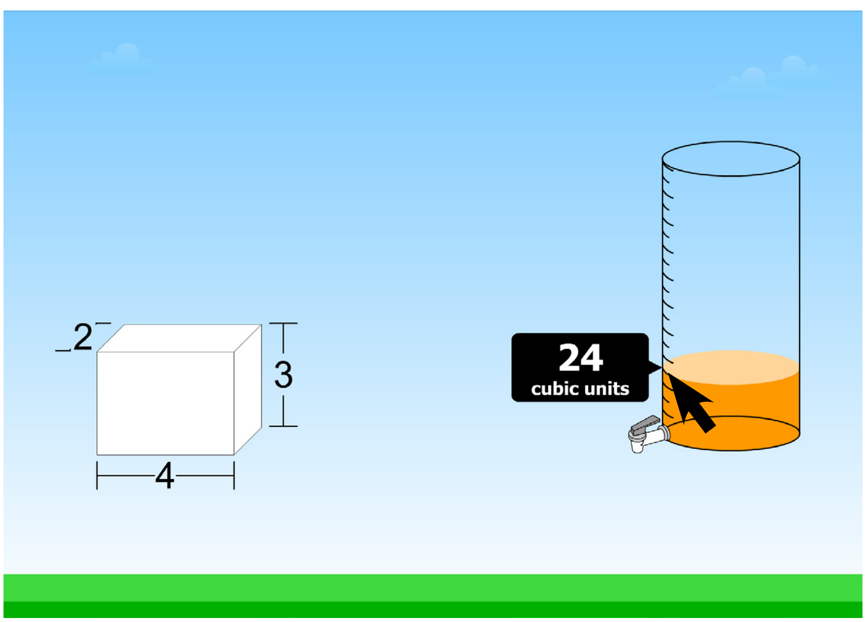
CLICK TO WATCH THE ANIMATION

In this puzzle, students begin by experiencing the concept of volume completely separate from the formula. They need to select the volume within the cylinder that matches the volume of the box. Here the student has selected 24.

Encouraging Learning from Mistakes

Game-based learning opportunities like this, that move away from familiar math drills and tests, put students in the mindset to explore! When students play digital games, they're used to trying things and learning from their mistakes until they reach their goal.

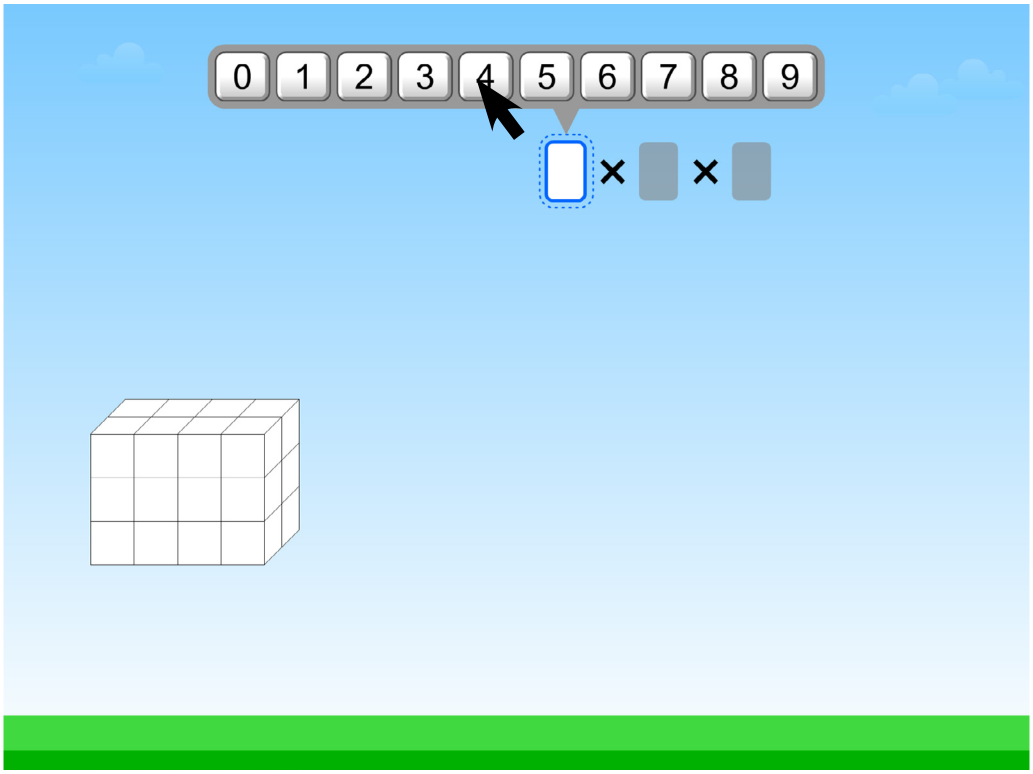




Here, students are still selecting the number of units in the cylinder needed to fill the box, but they are now given the numerical dimensions of the box, which continues to be represented in different ways. Eventually, they must find the volume without seeing the individual cubic units that make up the shape. These puzzles facilitate crucial connections between the concept of volume and the formula.

Building Conceptual Understanding

This sequence of volume puzzles intentionally builds conceptual understanding of three-dimensional volume. Students visualize and learn why the formula V=LWH works.



Here, students must understand the volume of a box as the product of its three dimensions. Building a deep understanding of the concept of volume ultimately equips students to solve volume problems in various different forms.

> *When students learn through visual approaches, mathematics changes for them, and they are given access to deep and new understandings.*

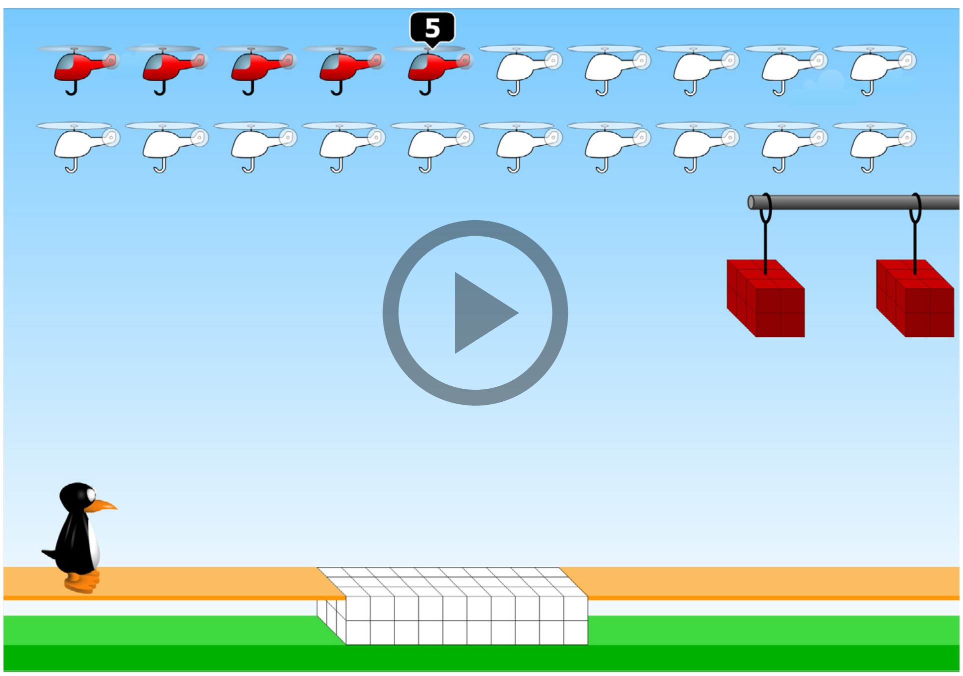
Jo Boaler
Seeing as Understanding: The Importance of Visual Mathematics for our Brain and Learning

Inspiring Creativity

ST Math puzzles ask students to internalize the problem, and forge a pathway toward the solution. There is often more than one way of solving the problem, which encourages creativity.

Facilitating Transference

Students eventually encounter the concept of volume in an entirely new context. This non-routine volume problem, asks the question: **How many sections of red blocks will fill the empty space in the pathway?** In this way, student understanding of volume expands as they apply proportional reasoning to deconstruct and construct three-dimensional shapes.



CLICK TO WATCH THE ANIMATION

In this non-routine volume problem, students need to figure out how many sections of red blocks will fill the pathway below. Students must select the number of helicopters, each of which pulls exactly one section of red blocks. In this way, student understanding of volume expands as they apply proportional reasoning to deconstruct and construct three-dimensional shapes.

MIND
Research Institute

Creators of the ST Math game-based learning software and the MathMINDs hands-on learning events, MIND Research Institute supports schools and communities in ensuring that all students are mathematically equipped to solve the world's most challenging problems.

REQUEST PROGRAM INFO

Connect With Us

About the Author:

Jessica Carlson is a Curriculum Expert at MIND Research Institute. She has taught middle school math and currently specializes in research-based, conceptual math education.

Editor: Heera Kang

Designer: Cassandra Cooper

Citations:
California Standards Test
Is Memorisation a Good Strategy for Learning Mathematics
Rigor & Relevance Framework
Seeing As Understanding: The Importance of Visual Learning for Our Brain and Learning
Smarter Balanced Assessment Consortium
Principles to Actions: Ensuring Mathematical Success for all