White Paper

February 18, 2011

Math and Science, Math and Science

ALAS Corporate Office
65 West Boston Post Road
Suite #200
Marlborough, MA 01752
Tel: (508) 486-4536
Fax: (508) 624-6565

Math and Science, Math and Science

In every school in America, parents, students, and teachers hear the mantra of “math and science, math and science” when contemplating what our students need to compete globally. This mantra stresses how imperative it is that we do something to increase the performance of all students in mathematics and science. We know from international assessments that we have a lot of work to do. One international test, the 2006 Program for International Student Assessment (PISA), made clear that 15-year olds in the United States were less proficient in mathematics than their peers in most other nations. The U.S. overall standing among 19 nations in both mathematics and science dropped from 8th in 2000 to 16th in 2006. The average mathematics score in the U.S. was lower than the average scores in 18 nations (out of 24) and higher than only 4 other countries—3 of which were developing economies. PISA tests students’ ability to apply what they have learned (i.e., explain answers in mathematical or scientific terms, use logical reasoning, and synthesize information). We know from our own national tests that our students are lacking in the critical skills of problem-solving and analytical reasoning. It doesn’t look any better for science. The PISA report reveals the number of countries scoring higher than the United States on the PISA science assessment rose from 6 in 2000 to 12 in 2006.

Another international test, the 2007 Trends in International Mathematics and Science Study (TIMSS), shows slightly different results for mathematics achievement and offers some encouragement (Martin et al., 2008). U.S. students’ 2007 average mathematics score was higher than their 1995 average score. The U.S. standing among selected countries also rose slightly, placing the United States near the median of selected nations in both grades. TIMSS tests closely follow the curriculums commonly taught in participating countries. However, results show that the average science scores of U.S. fourth and eighth graders had not changed measurably from the 1995 scores (Mullis et al., 2008). In fact, the U.S. position among other countries declined in fourth grade, but increased slightly in eighth grade. So while we wait to see signs of improvement, they are tempered by uneven growth.

Our national tests are somewhat more encouraging, although we still are not meeting even our most modest expectations for consistent improvement in student performance. One nine-year study, The Early Childhood Longitudinal Study, Kindergarten Class of 1999 (ECLS-K), offers valuable insights into the effect of preparation and cognition on mathematics achievement (NCES, 2010). This study followed nearly 8,000 students who first entered kindergarten in fall 1998 until most reached the eighth grade. Students in this study were tested in mathematics in grades 1, 3, 5, and 8 and in science in grades 3, 5, and 8. It is unique among national and international tests because it focused on the earliest time students are formally introduced to mathematical concepts. Furthermore, it allowed researchers to examine students’ performance in the context of other variables likely to influence learning, such as early home environment and parental characteristics.

What researchers found may explain the continued lag in student performance throughout the school years. The study revealed the need to target mathematics instruction more carefully in the early school years. Better preparation and instruction is needed for students as early as kindergarten. Researchers also identified those influences in the early home and learning environments that are so critical in preparing and supporting students for school and that, ultimately, increase their chance of success. These include time spent reading, income and educational backgrounds of the parents, access to effective and well-trained teachers, the child's ability to listen and concentrate, and the development of fine motor skills needed to write and draw (Corcoran and Evans, 2008; Grissmer and Eisemann, 2008; Magnuson et al., 2008).

Overall, study results demonstrated that kindergarten test scores of students in the ECLS-K, as well as demographic characteristics, were strong predictors of students' skill levels nine years later in 2007. Gaps generally grew through grade 3 and remained stable thereafter. For those groups experiencing more disadvantage or who were less prepared when they started school, the gaps were notable. This is particularly true for Hispanic and black students.

Table 1: Average Mathematics Scores of Students Followed from Kindergarten through Grade 8, by Student Characteristics: Fall 1998–Spring 2007

Student Characteristics	Fall 1998 Grade K	Spring 00 Grade 1	Spring 02 Grade 3	Spring 04 Grade 5	Spring 07 Grade 8
All students—mathematics score	26	62	99	123	139
Lowest quartile	17	46	78	101	120
Middle two quartiles	25	61	98	123	140
Highest quartile	38	79	123	144	157
Race/ethnicity					
White	29	66	106	129	145
Asian	30	65	105	133	149
Black	22	52	84	105	123
Hispanic	22	56	92	118	135
Mother's education in 1998					
Less than high school	20	51	82	106	123
High School diploma	24	58	93	116	133
Some college	26	63	101	125	142
Bachelor's degree	32	72	114	138	154

Sources: National Center for Education Statistics, *Early Childhood Longitudinal Study, Kindergarten Class, 1998–1999*, (ECLS-K), fall 1990 and spring 2000, 2002, 2004, and 2007. National Science Board, *Science and Engineering Indicators 2010*, p. 1–9.

Mathematical ability was measured on a single scale, with 0 being the lowest possible score and 174 the highest. On average, in 1998–99, the kindergarten cohort started with an average score of 26 points and gained 113 by the spring of eighth grade. During the early years of school, the gaps widened, but by grade 3 or 5 they began to stabilize or narrowed slightly, although overall growth also declined. It seems that students' relative achievement when starting school influenced their educational growth and eventual eighth-grade scores (National Science Board, Science and Engineering Indicators 2010, p. 1–8).

One predictor of student skill level is the mother's level of education. The gap between kindergartners whose mothers had no high school diploma versus those with a bachelor's degree was 12 points (20 versus 32). Race also mattered. White children scored 20 and Asian children 30 on the test given in fall 1998, compared with 22 for the black and Hispanic students.

Results from the National Assessment of Educational Progress (NAEP), another national test of math achievement, are more promising. Math scores increased among younger students through 2008, reflecting a trend begun in 1990. This test does not account for early preparation and only assesses the mathematical content knowledge by a given grade level. Nonetheless, both fourth and eighth graders improved from 1990 to 2007: the average fourth-grade score rose from 213 to 240; the average eighth grade score, from 263 to 281. These increases were shared by girls and boys; white, black, and Hispanic students; and students of varying socioeconomic status. Black fourth graders had the largest score increase, and white, black, and Hispanic eighth graders scored consistently higher in mathematics. Asians/Pacific Islanders' performance remained level after 2005, and American Indians/ Alaska Natives showed no change between 2000 (the first year with data available) and 2007. NAEP 2009 results show that the upward trend in fourth-grade mathematics scores has halted, that mathematics scores of eighth graders have continued to improve, and that score gaps among racial/ethnic groups are unchanged (National Center for Education Statistics, 2009a).

These tests suggest that some change in performance is occurring, but we cannot demonstrate the locus of that change, nor do we know what to attribute the change to. Overall, our performance is still weak whether we compare ourselves to ourselves or to our international counterparts. This is particularly troubling given the numerous reports over the last 20 years that identified science and mathematics achievement as vital to the economic viability of the United States. If the United States wants to remain a global leader in education, the environment, and technological innovation, we must outperform our global peers. Strong technical skills, quantitative literacy and reasoning, and innovation will provide U.S. citizens with the opportunities to make good wages and significant contributions not only to the global economy, but to their national and local communities as well.

More than Math and Science Education

Learning math and science is not enough—we need to encourage students to pursue careers in those fields. The President's Council of Advisors on Science and Technology (PCAST) 2010 Report to the President, Prepare and Inspire:K-12 Education in Science, Technology, Engineering, and Math (STEM) for America's Future, points out that in the 21st century, the country's need for a world-leading STEM workforce and a scientifically, mathematically, and technologically literate populace has become even greater. It will continue to grow—particularly as other nations continue to make rapid advances in science and technology. In the words of President Obama, "We must educate our children to compete in an age where knowledge is capital, and the marketplace is global" (p. vii). Yet, we know that in growing numbers our students are turning to professions other than those in the STEM fields. For example, African Americans, Hispanics, Native Americans, and women are seriously underrepresented in engineering. Furthermore, students are showing little interest in the math and science fields.

While there have been strides in better preparing students in math, science, and technology, more emphasis is needed. The No Child Left Behind Act (U.S. Public Law 2001) mandated testing and increased accountability for student performance, particularly in reading and mathematics. The American Recovery and Reinvestment Act of 2009 outlined improvements for the K–12 education system. The National Governors Association and Council of Chief State School Officers joined together in 2008 to develop clear, consistent standards for mathematics and English language arts education in grades K–12 that can be shared across states. To date, 36 states and the District of Columbia have adopted these standards. But it goes beyond even K–12 education. The 2005 report Rising above the Gathering Storm: Energizing and Employing America for a Brighter Economic Future and the subsequent 2010 report Rising Above The Gathering Storm, Revisited: Rapidly Approaching Category 5 identified gaps in the technical skills of our students and called for swift and immediate action to build the educational infrastructure that will prepare our students for a rapidly growing global economy.

Focusing first on K–12 education, the President in the fall of 2009 asked the President's Council of Advisors on Science and Technology (PCAST) to recommend the most important actions that his administration should take to ensure that the United States is a leader in STEM education in the coming decades. PCAST concluded that to improve STEM education, a strong focus on preparation as well as inspiration is needed to guide the implementation of their recommendations. This, coupled with a coherent strategy and strong leadership, will develop capacity for a comprehensive K–12 STEM education strategy.

Recommendations in the PCAST report to President Obama (2010) include:

- ✓ Support by the Federal government for the state-led effort to develop common standards in STEM subjects through professional development and the quality evaluation, administration, and improvement of ongoing assessments aligned to those standards.
- ✓ Recruit, prepare, and induct 100,000 STEM teachers for middle and high schools.
- ✓ Create a national STEM Master Teachers Corps that recognizes and rewards the nation's top 5 percent of all STEM teachers.
- ✓ Improve the quality of instructional materials available to teachers through technology, including the development of innovative technologies and technology platforms for learning, teaching, and assessment across all subjects and ages, as well as the development of effective, integrated whole-course materials for STEM education.
- ✓ Create opportunities that will inspire students both in and out of the classroom, individually and through group experiences.

These policy directives, combined with stronger standards set by the National Council of Teachers of Mathematics and the continued development of rigorous state-level assessments, will continue to strengthen the push for increased performance in mathematics and related fields.

Learning Approaches. Learning Math

Bringing together a coherent national agenda to improve mathematics and science education and integrating those disciplines into a more comprehensive STEM approach is a step in the right direction. It does not, however, address the fundamental issue of how to best teach mathematics and, more importantly, how to engage students in mathematics. Over the last several decades, the teaching of mathematics has seen one passing fad after another, with the introduction of New Math (Jones and Coxford, 1970) and discovery math (Bruner, 1966) in the 60s, both of which would meet with opposition and fail (Moon, 1986). The 1970s heralded in a renewed emphasis on back to basics and a focus on arithmetic. Students were taught each skill sequentially; learning techniques were by rote and memorization. Later that decade and into the 1980s, information processing theory and cognitive science began to dominate approaches to teaching mathematics. Problem-solving became a prominent focus of instruction (Hiebert and Behr, 1988; Marr, 1982; Paivio, 1986; Schoenfeld, 1987). Advances in cognitive theory led to learning more about how childrens' informal learning could connect to formal math instruction, conceptual understanding, and the development of constructivist theory, where new knowledge was thought to be "constructed" or invented by the learner (Papert and Harel, 1991).

Coupled with this was the seminal work of Howard Gardner in the early 1980s. In his book Frames of Mind: A Theory of Multiple Intelligences (1983), Gardner emphasized the connection between the brain and learning, with different kinds of intelligence having physiological locations in the brain (Gardner, 1983, 1998). Teachers became keenly interested in how to effectively teach an increasingly diverse student population by developing different strategies to address each of Gardner's eight, brain-based multiple intelligences—verbal-linguistic ; logical-mathematical; spatial ; bodily-kinesthetic; musical; naturalistic; interpersonal; intrapersonal, and existential.

The interest in brain-based multiple intelligences led to the new field of Brain-Based Learning (BBL), which draws from neurologists, biologists, psychologists, educators, and physicians. Research from the various fields is used to enhance an understanding of student learning and instruction. According to Jensen (1995), these strategies can also be used to enhance students' ability to learn in ways in which they feel most comfortable. He defines BBL as "learning in accordance with the way the brain is naturally designed to learn" (p.6). Other outgrowths of BBL inspired such instructional approaches as differentiated instruction (Tomlinson 1999); multisensory approaches to reading, writing, and math (FastForWord, Wilson Reading System); learning style approaches (Connell, 2005; Frender, 1994); left brain, right brain strategies (Connell, 2005; Gunzelmann and Connell, 2006; Springer and Deutsch, 2001); and the more recent emphasis on spatial-temporal (ST) learning (Rutherford, et al., 2010).

Many instructional strategies have been developed, tried, and some tested, but despite their effectiveness, the quality of the teacher delivering the content continues to be a critical factor. The extent to which teachers are well-qualified to teach mathematics is not well documented. No Child Left Behind mandated that schools and school districts hire only highly qualified teachers, defining "highly qualified" in terms of state certification, a minimum of a bachelor's degree, and demonstrated subject area competence. However, faced with increasingly difficult school environments in which to teach, safety issues, overcrowding, and growing numbers of English Language Learners, it is difficult to attract the best and brightest teachers to the schools most in need. In many of our poorer and urban school districts, a number of teachers have only emergency, provisional, or temporary licenses, and many teachers lack subject area content knowledge for the classes in which they are being asked to teach.

The most recent data on the highest degree and certification status attained by teachers indicate that virtually all of them had at least a bachelor's degree, nearly half had a master's or higher degree, and a majority held a regular or advanced teaching certificate (NSB, 2008). Most states require teachers of mathematics and science at the high school level to have a degree or certification in their subject area; however, state laws and regulations vary regarding the preparation of middle school teachers. Elementary school teachers typically have multiple subject credentials and are considered qualified to teach elementary school mathematics and science, although they typically lack a degree that is specific to mathematics.

Access to better qualified teachers also varies among racial groups. In general, black and Hispanic students from less-educated and low-income families and students with low levels of prior achievement had lower access to teachers who were highly educated, fully certified, and more experienced in their subject field than their counterparts. For example, fifth-grade black and Hispanic students were less likely than their white peers to be taught mathematics by teachers with a master's or advanced degree (39% and 42% vs. 51%, respectively), a regular or advanced teaching certificate (86% and 85% vs. 92%, respectively), and more than three years of experience in teaching the fifth grade (48% and 58% vs. 68%, respectively). Students living in low-income families were less likely than their peers from higher-income families to be taught mathematics by teachers with a master's or advanced degree (35% vs. 50%, respectively). Among fifth graders, a third of those in the lowest achievement quartile in grade 3 were taught mathematics by in-field teachers. In contrast, 41% of fifth graders in the top achievement quartile in grade 3 had such teachers (NSB, 2010).

The issues of teaching and learning mathematics are very complex, and more research is needed to understand what works and for what populations. Teacher preparation and professional development continues to be an area that needs examination. Although Federal mandates require "highly qualified" teachers, each state sets its own standards for teacher preparation and certification, resulting in a lack of consistency across states. A movement toward individualized, computer-based instruction transfers the responsibility of learning to the individual while allowing ample formative assessment and real-time correction. This approach enables learners to master content while relying less upon the content specialty of the teacher. One such individualized learning system, based on the BBL movement, is the increasingly popular spatial-temporal (ST) learning platform.

Spatial–Temporal (ST) Learning: A Promising Approach to Mathematics Instruction

Spatial-Temporal (ST) learning has led to the development of a number of computer-based instructional programs. Researchers demonstrated that music can enhance how we think, reason, and create (Leng and Shaw, 1991). Of more interest educationally is a study showing that preschool children who received piano keyboard lessons for six months improved their performance dramatically on an ST reasoning task, with the effect lasting for days. Grandin, et al. (1998) suggested that certain math and science concepts known to be difficult to teach can be learned using ST reasoning methods, especially at an early age. They further noted that music instruction can enhance the "hardware" in the brain for ST reasoning.

These researchers distinguished between two types of reasoning: language-analytic (LA) and spatial-temporal (ST). They believe that learners typically move between both, using language-analytic reasoning modes for solving problems or equations and the spatial-temporal (ST) mode for more complex tasks, which may include mental images and thinking "several moves ahead". In particular, researchers argued that ST reasoning is crucial in learning math and, in particular, for proportional reasoning, which has been shown to be difficult to teach U.S. school children using traditional language-analytic methods (Karplus et al., 1983; cited in Grandin et al., 1998).

Spatial cognition is particularly valuable because it relates directly to preparation in the STEM fields. We know from our international tests that students are performing poorly on those mathematic skills related to spatial representation, like measurement and geometry (Ginsburg et al., 2005). We also know from our national tests, NAEP and ECLS, that the items where students tended to score particularly low were fractions, rate, and measurement—all items that the development ST teaching techniques would address. Additionally, many STEM organizations have recognized the need for encouraging spatial reasoning among students and in the classroom. The Committee on Support for Thinking Spatially (2006) specifically mentions that there is a strong need to develop spatial thinking by giving students the appropriate tools and methods to develop this capacity.

The Mind Research Institute

The Mind Research Institute has developed several modules that use an ST approach to learning mathematics, and recent research has demonstrated that these yield positive results in math performance. MIND's education programs teach all children, regardless of socioeconomic or cultural background, how to think, reason, and create mathematically. These programs have been designed to meet the major challenges facing public education—language barriers and diversity—and to help improve teacher efficacy and efficiency in the classroom. MIND's unique math education approach engages the learner's spatial-temporal (ST) abilities to explain, understand, and solve multi-step problems The Institute is dedicated to finding ways to continually improve student math performance, which will ultimately lead to a more productive educational workforce. The MIND programs are meeting the President's challenge to better prepare our nation's school children in the STEM fields.

Born out of neuroscience research at the University of California, Irvine, MIND's unique approach accesses the brain's innate ST reasoning ability. This ability allows the brain to hold visual, mental representations in short-term memory and to evolve them both in space and time, thinking multiple steps ahead. MIND's approach consists of language-independent, animated representations of math concepts delivered via computer software games. Self-paced and self-motivating, the Spatial-Temporal (ST) Math® programs give students immediate, instructive feedback and deepen problem-solving and reasoning skills. Students gain confidence in their skills and a desire to advance their mathematical knowledge. Additionally, MIND's comprehensive courseware aligns to state standards. Their products address the need for early intervention and learning in their series ST Math®: K–5, STMath®:Fluency, ST Math + Music®: K–5, and, for older students, ST Math®: Secondary Intervention and Algebra Readiness.

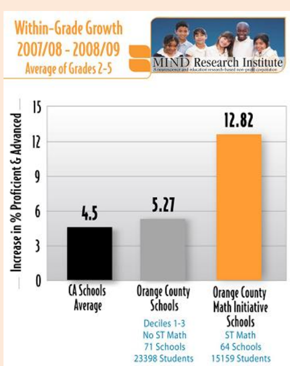
ORANGE COUNTY MATH INITIATIVE

2008-2009 CST Math Results

Students in grades using ST Math in Orange County Math Initiative schools, selected dominantly from the lowest 3 deciles of statewide performance, were tested on the California Standards Test in Math at the end of the school year 2008/09.

Within-grade growth compares the percentage proficient or better in 2008/09 to the percentage proficient or better in 2007/08. The 2008/09 **within-grade average growth** for all grades (2–5) at the 64 Orange County schools using ST Mathschools **was 12.82 points.**

Seventy-one **schools not using ST Math scored only 5.27 within grade growth.** A further comparison shows that the 2008/09 **within-grade average growth for the state of California overall for was 4.5 points.**



Grades using ST Math realized greater within-grade average growth at each grade level.

MIND is committed to better understanding how its curriculum enhances student performance. In 1999, the first pilot study of its program at an elementary school in South Central Los Angeles (Graziano et al., 1999) revealed that participating students scored in the 65th percentile on the Stanford 9 Math Test (used in California prior to the California Standards Test, CST). Non-participants at the same school scored at the 36th percentile. Further demonstrations of students' increased math proficiency, despite their previous performance levels, continue MIND's track record. What is particularly unique about MIND's approach is that it appeals to all students—even students who struggle with the English language can improve their math scores significantly because language is not required to navigate through the curriculum.

Thus far, research supports the viability of ST instruction. As noted in the studies above, students are experiencing significant gains in mathematics as evidenced by growth on state tests. Rutherford, et al. (2010) reports that earlier quasi-experimental and smaller-scale experimental studies showed positive effects from the training of spatial thinking on mathematics outcomes, and comparable effects of spatial training have also been found in college populations.

How Does Spatial-Temporal (ST) Math Instruction Work and for Whom Does it Work Best?

The functional beauty of ST Math instruction is that language is not required. For many teachers of students who are English language learners (ELLs) or Limited English Proficient (LEPs), this removes a language barrier. These students can begin to learn math without having to filter it through language instruction. As

Rutherford, et al. (2010) note, ST Math may allow ELLs to master mathematical concepts without simultaneously having to master English-related peculiarities of math learning. ST Math gives them a scaffolded introduction to math symbols and language once a conceptual basis is established. It provides a standards-based, grade-level curriculum to non-English speakers, better preparing them for grade-level assessments, like the CSTs, setting a solid foundation on which future math learning can be built. The approach also shows promise with underrepresented groups who are not achieving in math or science fields. These groups typically are women, blacks, Hispanics, and Native Americans. While research is lacking that can demonstrate that ST Math can help a variety of students with special learning needs or who typically are underachievers, early indications seem promising.

So what makes this approach so compelling? The program uses mathematical shapes to make mathematics concepts more accessible, as opposed to the more conventional language and symbol-based approaches commonly used in mathematics instruction. This is particularly beneficial because it tends to increase a student's level of engagement with the material, making it more enjoyable to learn.

The simple platform is a one-to-one, interactive, animated learning environment in which a student works at his or her own pace. The learner receives feedback to correct and incorrect solutions through animation. Since students today are keenly astute at manipulating video games and using interactive technology, they find the ST Math approach appealing and engaging. Constant feedback gives the learner valuable information to guide his or her progress toward the goal of self-regulated learning (Metcalfe and Kornell, 2007). A video describing the differences between the ST approach to math and conventional math can be viewed at the MIND Research website, www.mindresearch.net/video/demo.html.

Conclusion

New approaches to mathematics instruction appear to be strengthening student achievement. It is particularly encouraging when courseware can also address some of the educational concerns, such as individual learning differences, language barriers, and diversity, that are not easily addressed in today's classrooms. Spatial visualization may be the key to unlocking students' conceptualization of fundamental mathematical principles from which to build mathematical understanding, computational fluency, and problem-solving skills. The self-paced, self-correcting animated formats may also engage and motivate students in their studies and inspire them to see mathematics as a powerful tool for today's complex environment.

References

Bruner, J. (1966). *Toward a theory of instruction*. New York: W.W. Norton and Company.

Committee on Support for Thinking Spatially (2006). *Learning to Think Spatially: GIS as a Support System in the K-12 Curriculum*. The National Academies Press.

Connell, D. (2005). *Brain-based strategies to reach every learner*. New York:Scholastic.

Corcoran, Sean P. and William N. Evans (2008). "The Role of Inequality in Teacher Quality," in *Steady Gains and Stalled Progress: Inequality and the Black-White Test Score Gap*, ed. Katherine Magnuson and Jane Waldfogel, Russell Sage Foundation.

Fast ForWord. Fast ForWord Reading Series and Educational Software. Oakland, CA: Scientific Learning Corp.

Frender, G. (1994). *Teaching for learning success*. Nashville, TN: Incentive Publications.

Gardner, H. (1983). *Frames of mind: The theory of multiple intelligences*. New York, NY: Basic Books.

Gardner, H. (1998). Keynote presentation, Fourth International Teaching for Intelligence Conference: New York, April 23.

Ginsburg, A., Cooke, G., Leinwand, S., Noell, J., and Pollock, E. (2005). *Reassessing U.S. International Mathematics Performance: New Findings from the 2003 TIMSS and PISA*. Washington, DC: The American Institutes for Research.

Grandin, T., Peterson, M. and Shaw, G., L. (1998). Spatial-temporal versus language-analytic reasoning: The role of music training. *Arts Education Policy Review*, 99(5): 11-15.

Graziano, A.B., Peterson, M., and Shaw, G.L. (1999). Enhanced learning of proportional math through music training and spatial temporal training. *Neurological Research*, 21(2):139-52.

Grissmer, David, and Eiseman, B. (2008). Can Gaps in the Quality of Early Environments and Non-cognitive Skills Help Explain Persisting Black-White Achievement Gaps? In *Stalled Progress: Inequality and the Black-White Test Score Gap*. Jane Waldfogel and Katherine Magnuson, eds, Russell Sage.

Gunzelmann, B., and D. Connell. (2006). The new gender gap: Psychological, neurobiological, and educational Perspectives. *Educational Horizons*, 84 (2),

Hiebert, J., and Behr, M. (1988). *Number concepts and operations in the middle grades*. Hillsdale, NJ: LEA.

Jensen, E. (1995). *Brain-based Learning: The New Science of Teaching and Training*. Thousand Oaks, CA: Corwin Press. [Revised in 2000].

Jones, P., and Coxford, A. (1970). Mathematics in the evolving schools. In NCTM (Ed.), *A history of mathematics education in the United States and Canada* (pp. 11-92). Washington, DC: National Council of Teachers of Mathematics.

R. Karplus, S. Pulos, and K. Stage. (1983). Early adolescents' proportional reasoning on `rate' problems. *Educational Studies in Math* 14(219).

Leng, X. and Shaw, G. L. (1991). Toward a neural theory of higher brain function using music as a window. *Concepts Neurosci*, 2(229).

Magnuson, K., Rosenbaum, D. and Waldfogel, J. (2008). Inequality and Black-White Achievement Trends for 9-Year-Olds in the NAEP. In K. Magnuson and J. Waldfogel (eds.), *Steady Gains and Stalled Progress: Inequality in the Black- White Test Score Gap* (33-65). New York, Russell Sage.

Marr, D. (1982). *Vision: A Computational Investigation into the Human Representation and Processing of Visual Information*. W. H. Freeman and Co., San Francisco.

Martin, M.O., Mullis, I.V.S., and Foy, P. (2008). *TIMSS 2007 International Science Report: Findings from IEA's Trends in International Mathematics and Science Study at the Fourth and Eighth Grades*. Chestnut Hill, MA: TIMSS and PIRLS International Study Center, Boston College.

Metcalf, Janet, and Kornell, Kate. (2007). Principles of cognitive science in education: The effects of generation, errors, and feedback. *Psychonomic Bulletin and Review* 2007, 14 (2), 225-229.

Moon, B. (1986). *The 'new maths' curriculum controversy: An international story*. Philadephia, PA: The Falmer Press.

Mullis, I.V.S., Martin, M.O., and Foy, P. (2008). *TIMSS 2007 International Mathematics Report: Findings from IEA's Trends in International Mathematics and Science Study at the Fourth and Eighth Grades.* Chestnut Hill, MA: TIMSS and PIRLS International Study Center, Boston College.

National Academy of Sciences, National Academy of Engineering, Institute of Medicine. (2007). *Rising above the Gathering Storm: Energizing and Employing America for a Brighter Economic Future*, National Academies Press [The initial report release was in 2005, with the final, edited book issued in 2007]

National Academy of Sciences, National Academy of Engineering, Institute of Medicine. (2010). *Rising Above the Gathering Storm, Revisited: Rapidly Approaching Category 5*, National Academies Press. http://www.nap.edu

National Center for Education Statistics (NCES). (2009a). *The Nation's Report Card: Mathematics 2009*. NCES2010-451. Washington, DC.

National Center for Education Statistics (NCES). (2009b). *The Condition of Education 2009*. NCES 2009-081. Washington, DC.

National Center for Education Statistics (NCES). (2010). *Early Childhood Longitudinal Study, Kindergarten Class of 1998-99 (ECLS-K) Kindergarten Through Fifth Grade Approaches to Learning and Self-Description Questionnaire (SDQ) Items and Public-Use Data Files*. Publ. #NCES 2010070.

National Science Board. (2008). *Science and Engineering Indicators 2008*. Arlington, VA: National Science Foundation. 1: 23-27 Available at: http://www.nsf.gov/statistics/seind08/

National Science Board. (2010). *Science and Engineering Indicators 2010*. Arlington, VA: National Science Foundation. Available at: http://www.nsf.gov/statistics/seind10/

Paivio, A. (1986). *Mental representations: A dual coding approach*. New York: Oxford Press.

Papert, S. and Harel, I. (eds). (1991). *Constructionism: research reports and essays 1985-1990 by the Epistemology and Learning Research Group*, the Media Lab, Massachusetts Institute of Technology, Ablex Pub. Corp, Norwood, NJ.

President's Council of Advisors on Science and Technology. (2010). *Prepare and Inspire: K-12 Education in Science, Technology, Engineering, and Math (Stem) for America's Future*. Report to the President. Retrieved from http://www.whitehouse.gov/sites/default/files/microsites/ostp/pcast-stemed-report.pdf

Programme for International Student Assessment. (2007). *PISA 2006: Science Competencies for Tomorrow's World*. PISA: pp. 383

Rutherford, T., Kibrick, M., Burchinal, M., Richland, L., Conley, A., Osborne, K., Schenider, S., Duran, L., Coulson, A., Antenore, A., Daniels, A. and Martinez, M. E. (2010, May). *Spatial temporal mathematics at scale: An innovative and fully developed paradigm to boost math achievement among all learners*, a paper presented at the annual convention of the American Educational Research Association, Denver, CO.

Schoenfeld, A. (1987). Cognitive science and mathematics education overview. In A.Schoenfeld (Ed). *Cognitive science and mathematics education* (pp.1-32). Hillsdale, NJ: LEA.

Silver, E. (1987). Cognitive theory and research. In A. Schoenfeld (Ed.), *Cognitive science and mathematics education* (pp.33-60). Hillsdale, NJ: LEA.

Springer, Sally P., and Deutsch, G. (2001). *Left Brain, Right Brain: Perspectives From Cognitive Neuroscience*. Worth Publishers, Incorporated.

Tomlinson, C. (1999). *The differentiated classroom: Responding to the needs of all learners*. Alexandria, Va.: Association for Supervision and Curriculum Development.

U.S. Public Law 107–110, 107th Congress. Jan. 8, 2002. No Child Left Behind Act of 2001.

Wilson Reading System. Wilson Language Training. Oxford, Mass.

This paper was written and produced with support from Mind Research and the Association of Latino Administrators and Superintendents (ALAS).